

Polarized Λ hyperon production in Semi-inclusive deep inelastic scattering off an unpolarized nucleon target

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We study the production of polarized Λ hyperon in semi-inclusive deep inelastic scattering off an unpolarized target. We include the cases in which the Λ hyperon is longitudinally polarized or transversely polarized, and in which the lepton beam is unpolarized or longitudinally polarized. Within the framework of the transverse momentum dependent factorization, we take into account the complete decomposition of the parton correlator for fragmentation up to twist-3. We present the cross section of the process to order $1/Q$. The expressions of the polarized structure functions, which may give rise to various spin asymmetries, are also given.

PACS numbers: 13.60.Hb, 13.87.Fh, 13.88.+e

I. INTRODUCTION

Understanding the spin structure of the Λ hyperon is one of the most challenging problems in spin physics. The study of the Λ polarization dates from 1970s when Fermilab [1] conducted the pioneer measurement in hadronic collision at 300 GeV. The measurement of polarization phenomena of Λ hyperon production in semi-inclusive deep inelastic scattering (SIDIS) can provide further information about the spin structure of Λ hyperon and spin-dependent dynamics of fragmentation region, such as the mechanism of spin transfer from outgoing struck quark to a Λ hyperon [2–6]. The polarization of Λ hyperon can be measured by looking at the angle distribution of decay $\Lambda \rightarrow p\pi$, since the decayed proton will preserve the polarization information of Λ hyperon.

The longitudinal polarization of Λ hyperon can be generated in SIDIS by a longitudinally polarized beam off an unpolarized target. The angular momentum conservation indicates that outgoing quark has the same spin orientation as lepton beam and the polarized quark could fragment into a Λ hyperon and transfer its polarization in process. The longitudinal spin transfer has been measured the HERMES collaboration [7–12] and the COMPASS collaboration [13–15]. The production of transverse polarized Λ hyperon in lepton-nucleon scattering has also been proposed as a useful tool to study its spin structure. Although the transversely polarized Λ hyperon production has been measured in hadron collisions [16–19] with different beams, very little experimental information about Λ polarization is available from lepton production [20–23]. In SIDIS, polarized Λ production is related to quark polarization inside the nucleons as well as the hadronization process in final state.

In this work, we study the semi-inclusive lepton production of longitudinally or transversely polarized Λ hyperon: $\ell + N \rightarrow \ell' + \Lambda + X$, in which a lepton beam (unpolarized or longitudinal polarized) scatters off an unpolarized nucleon target. To this end, we consider the decomposition of the quark correlation function to the transverse momentum dependent (TMD) parton distribution functions (PDFs) and fragmentation functions (FFs), up to the subleading order of the $1/Q$ expansion. Within the TMD factorization framework, we compute the parton-model results of the cross section which is differential to the transverse momentum of the Λ hyperon. Particularly, we pay more attention to the T-odd PDFs and FFs, since they play important role on various azimuthal or spin asymmetries in SIDIS. As shown in Refs. [24–26], the presence of the direction of the Wilson line in the decomposition of the parton correlation function will introduce several twist-3 T-odd functions that has not been considered in previous studies [27, 28]. We will also include these functions to compute the Λ -spin dependent structure functions.

The paper is organized as follows. In Sec. II, we review the decomposition of the parton correlation functions up to twist-3 level. We then perform the calculation of the hadronic tensor for Λ hyperon production in SIDIS with an unpolarized target. In Sec. III we present the cross section of the process and provide the spin-dependent structure functions. Finally, we summarize our results and give conclusions in Sec. IV.

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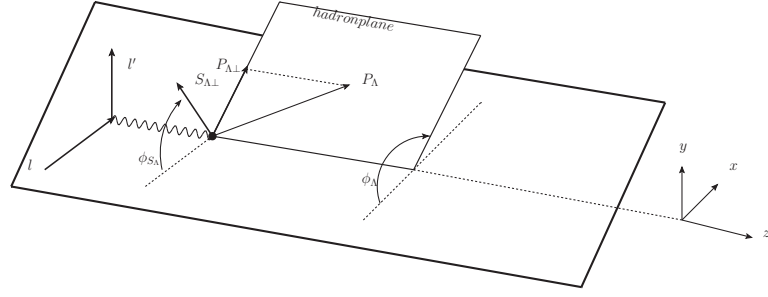


FIG. 1: Definition of azimuthal angles for SIDIS in the $\gamma^* N$ collinear frame [29], the lepton plane is determined by l, l' . $P_{\Lambda\perp}$ and $S_{\Lambda\perp}$ are transverse parts of P_Λ and S_h with respect to the photon momentum.

II. FORMALISM OF THE CALCULATION

We consider the Λ hyperon production in SIDIS

$$\ell(l) + N(P) \rightarrow \ell'(l') + \Lambda(P_\Lambda) + X, \quad (1)$$

where we use l, l', P and P_Λ to denote the momentum of the incoming lepton beam, outgoing lepton, the nucleon target N and the Λ hyperon, respectively. The momentum of the exchanged virtual photon is defined as $q = l - l'$ and $Q^2 = -q^2$. We also define the masses of nucleon and Λ hyperon as M and M_Λ . To express the cross section, we introduce the invariant variables

$$x = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_\Lambda}{2P \cdot q}, \quad \gamma = \frac{2Mx}{Q}. \quad (2)$$

Following the ‘‘Trento convention’’ [29], we define the azimuthal angles ϕ_h of the detected Λ hyperon between transverse momentum part and lepton plane as

$$\cos \phi_\Lambda = -\frac{l_\mu P_{\Lambda\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{\Lambda\perp}^2}}, \quad \sin \phi_\Lambda = -\frac{l_\mu P_{\Lambda\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{\Lambda\perp}^2}}, \quad (3)$$

where $\ell_\perp^\mu = g_\perp^{\mu\nu} \ell_\nu$ and $P_{\Lambda\perp}^\mu = g_\perp^{\mu\nu} P_{\Lambda\nu}$. We introduce perpendicular projection tensors

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} + \hat{z}^\mu \hat{z}^\nu - \hat{t}^\mu \hat{t}^\nu, \quad (4)$$

$$\epsilon_\perp^{\mu\nu} \equiv -\epsilon^{\mu\nu\rho\sigma} \hat{t}_\rho \hat{z}_\sigma, \quad (5)$$

with nonzero components $g_\perp^{11} = g_\perp^{22} = -1$ and $\epsilon_\perp^{12} = -\epsilon_\perp^{21} = 1$. It is convenient to expand the leptonic and hadronic tensor with respect to the virtual photon direction. The two normalized vectors \hat{t} and \hat{z} are

$$\hat{t}^\mu = \frac{xP^\mu}{Q} + \frac{q^\mu}{Q}, \quad \hat{z}^\mu = -\frac{q^\mu}{Q} = -\hat{q}^\mu. \quad (6)$$

We decompose the covariant spin vector S_Λ of the Λ hyperon as

$$S_\Lambda^\mu = S_{\Lambda\parallel} \frac{P_\Lambda^\mu - q^\mu M_\Lambda^2 / (P_\Lambda \cdot q)}{M_\Lambda \sqrt{1 + \gamma^2}} + S_{\Lambda\perp}^\mu. \quad (7)$$

The azimuthal angle ϕ_{S_Λ} relevant for specifying the polarization of Λ hyperon is obtained from Eq. (3) by the replacement: $P_\Lambda \rightarrow S_\Lambda$. In our case, the target nucleon is unpolarized and the detected Λ hyperon is transversely or longitudinally polarized.

The cross section of SIDIS can be expressed as the contraction of the hadronic tensor and the leptonic tensor:

$$\frac{d\sigma}{dx dy dz d\phi_\Lambda d\psi dP_{\Lambda\perp}^2} = \frac{\alpha^2 y}{8Q^4 z} L_{\mu\nu} 2M W^{\mu\nu}, \quad (8)$$

where the angle ψ is the azimuthal angle of the outgoing lepton ℓ' around beam axis with respect to an arbitrary fixed direction, and we can choose it to be the transverse polarization direction of the Λ hyperon. The expression of the cross section can be simplified with ϕ_{S_Λ} instead of ψ . In deep inelastic kinematics, one has $d\psi \approx d\phi_{S_h}$ [30, 31].

In the $\gamma^* N$ collinear frame [27], the lepton momentum can be expanded into \hat{t} , \hat{z} and the perpendicular components. Thus, we have the leptonic tensor (neglecting the lepton mass):

$$\begin{aligned} L^{\mu\nu} = & \frac{Q^2}{y^2} \left[-2(1-y + \frac{1}{2}y^2)g_{\perp}^{\mu\nu} + 4(1-y)\hat{t}^{\mu}\hat{t}^{\nu} \right. \\ & + 4(1-y)(\hat{x}^{\mu}\hat{x}^{\nu} + \frac{1}{2}g_{\perp}^{\mu\nu}) + 2(2-y)\sqrt{1-y}\hat{t}^{\{\mu}\hat{x}^{\nu\}} \\ & \left. - i\lambda y(y-2)\epsilon_{\perp}^{\mu\nu} - 2i\lambda y\sqrt{1-y}\hat{t}^{[\mu}\epsilon_{\perp}^{\nu]\rho}\hat{x}_{\rho} \right], \end{aligned} \quad (9)$$

where $\hat{x}^{\mu} = l_{\perp}^{\mu}/|l_{\perp}|$ is a unit vector in perpendicular direction. The lepton helicity is denoted by λ .

The hadronic tensor in SIDIS is defined as:

$$2MW^{\mu\nu} = \frac{1}{(2\pi)^3} \sum_X \int \frac{d^3 P_X}{2P_X^0} \delta^{(4)}(q + P - P_X - P_{\Lambda}) \langle P | J^{\mu}(0) | P_{\Lambda} S_{\Lambda}; P_X \rangle \langle P_{\Lambda} S_{\Lambda}; P_X | J^{\nu}(0) | P \rangle, \quad (10)$$

where $J^{\mu}(\xi)$ is the electromagnetic current. It is understood that the sum \sum_X is also over the polarization of undetected hadrons in final state.

In the factorization framework [32–35], the cross section of SIDIS can be written as the convolution of the lepton-quark scattering process (the hard part) and non-perturbative TMDs (the soft part) [36–40]. At the tree level, the hadronic tensor can be factorized in terms of various TMD PDFs and FFs up to sub-leading twist in the sense of $1/Q$ expansion. Therefore, the hadronic tensor can be obtained from the diagrams shown in Fig. 2. Here, Fig. 2a only involves quark-quark matrix elements, and Fig. 2b and Fig. 2c involve quark-gluon-quark matrix elements. The “h.c.” represents the diagrams hermitian conjugate to Fig. 2b and Fig. 2c, with gluon attaching to the other side of the final state cut. Up to $\mathcal{O}(1/Q)$, the corresponding contributions of the hadronic tensor can be expressed as [25, 27, 41, 42]

$$\begin{aligned} 2MW^{\mu\nu} = & \sum_a e_a^2 \int d^2 p_T d^2 k_T \delta^2(p_T - k_T + q_T) \text{Tr} \left\{ \Phi^a(x, p_T) \gamma^{\mu} \Delta^a(z, k_T) \gamma^{\nu} \right. \\ & \left. - \gamma^{\alpha} \frac{\not{p}_+}{Q\sqrt{2}} \gamma^{\nu} \tilde{\Phi}_{A\alpha}^a(x, p_T) \gamma^{\mu} \Delta^a(z, k_T) - \gamma^{\alpha} \frac{\not{p}_-}{Q\sqrt{2}} \gamma^{\mu} \tilde{\Delta}_{A\alpha}^a(z, k_T) \gamma^{\nu} \Phi^a(x, p_T) - h.c. \right\}. \end{aligned} \quad (11)$$

In Eq. (11), the terms with n_+ and n_- arise from fermion propagators in the quark-lepton scattering part with corrections of order $1/Q$. [42, 43]

Note that here we decompose the spin and momentum vectors using two light-like vectors n_+ and n_- in light-cone coordinate, in which the transverse direction is defined in the ΛN collinear frame (T-vectors). Particularly, P and P_{Λ} have no transverse momentum part, and they can be decomposed as

$$P^{\mu} = P^+ n_+^{\mu} + \frac{M^2}{2P^+} n_-^{\mu}, \quad P_{\Lambda}^{\mu} = P_{\Lambda}^- n_-^{\mu} + \frac{M_{\Lambda}^2}{2P_{\Lambda}^-} n_+^{\mu}, \quad (12)$$

with n_+ and n_- can be expressed in terms of \hat{t} , \hat{z} and the transverse component q_T ,

$$n_+^{\mu} = \frac{1}{\sqrt{2}}(\hat{t}^{\mu} + \hat{z}^{\mu}), \quad n_-^{\mu} = \frac{1}{\sqrt{2}}(\hat{t}^{\mu} - \hat{z}^{\mu} - 2\frac{q_T^{\mu}}{Q}). \quad (13)$$

Thus the relation between the two bases of the transverse vectors can be obtained from the following expression [27, 42]:

$$g_T^{\mu\nu} = g_{\perp}^{\mu\nu} - \frac{Q_T}{Q} \hat{q}^{\{\mu} \hat{x}^{\nu\}} + \frac{Q_T}{Q} \hat{t}^{\{\mu} \hat{x}^{\nu\}}. \quad (14)$$

The decomposition of the spin vector S_{Λ} has the form

$$S_{\Lambda}^{\mu} = S_{\Lambda L} \frac{(P_{\Lambda} \cdot n_+)n_-^{\mu} - (P_{\Lambda} \cdot n_-)n_+^{\mu}}{M_{\Lambda}} + S_{\Lambda T}^{\mu}, \quad (15)$$

To construct the hadronic tensor, we start from the general structure of the correlation functions [44] shown in Eq. (11), which are Φ for the quark distributions, Δ for the quark fragmentation, and quark-gluon-quark correlator $\tilde{\Phi}_A$ and $\tilde{\Delta}_A$.

The quark-quark distribution correlation function for unpolarized nucleon in SIDIS is defined as

$$\Phi(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P | \bar{\psi}(0) \mathcal{U}_{(0,+\infty)}^{n_-} \mathcal{U}_{(+\infty,\xi)}^{n_-} \psi(\xi) | P \rangle \Big|_{\xi^+=0}. \quad (16)$$

In this correlator, the Wilson lines are given as

$$\mathcal{U}_{(+\infty,\xi)}^{n_-} \equiv \mathcal{U}_{(\infty T, \xi_T; +\infty^\pm)}^T \mathcal{U}_{(+\infty^-, \xi^-; \xi_T)}^{n_-}, \quad (17)$$

$$\mathcal{U}_{(0,+\infty)}^{n_-} \equiv \mathcal{U}_{(\xi^-, +\infty^-, 0_T)}^{n_-} \mathcal{U}_{(0_T, \infty T; +\infty^-)}^T, \quad (18)$$

where the superscript n_- of \mathcal{U} indicates a Wilson line running along the minus direction in SIDIS. Detailed definitions of the Wilson lines for TMDs can be found in Refs. [41, 45, 46]. Particularly, the definitions of Wilson lines for the correlation functions can differ in different processes [47–50]. For instance, all occurrences of ∞^- in Wilson line in SIDIS should be replaced by $-\infty^-$ in Drell-Yan process.

The quark-quark correlator for an unpolarized nucleon can be decomposed as [25, 26, 41]

$$\begin{aligned} \Phi(x, p_T) = & \frac{1}{2} \left\{ f_1 \not{p}_+ + i h_1^\perp \frac{[\not{p}_T, \not{p}_+]}{2M} \right\} \\ & + \frac{M}{2P^+} \left\{ e + f^\perp \frac{\not{p}_T}{M} - g^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma} \gamma_\rho p_{T\sigma}}{M} + i h \frac{[\not{p}_+, \not{p}_-]}{2} \right\}, \end{aligned} \quad (19)$$

where we limit the expression to the leading and subleading term in $1/Q$ expansion.

The fragmentation correlator Δ is defined as [42, 51–53]

$$\Delta(z, k_T) = \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 | \mathcal{U}_{(\infty,\xi)}^{n_+} \psi(\xi) | P_\Lambda, S_\Lambda; X \rangle \langle P_\Lambda, S_\Lambda; X | \bar{\psi}(0) \mathcal{U}_{(0,+\infty)}^{n_+} | 0 \rangle \Big|_{\xi^-=0}, \quad (20)$$

with $k^- = \frac{P_-}{z}$, where the momentum fraction z in fragmentation functions coincide with the variable defined in Eq. (2). An similar remark holds for the Bjorken variable x in the definition of distribution functions.

Up to twist-3 level, a complete parameterization of the fragmentation correlator Δ complying with hermiticity and parity constraints, can be given as [44]

$$\begin{aligned} \Delta(z, k_T) = & \frac{1}{2} \left\{ D_1 \not{p}_- + D_{1T}^\perp \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T\sigma}}{M_\Lambda} + G_{1s} \gamma_5 \not{p}_- \right. \\ & + H_{1T} \frac{[\not{S}_{\Lambda T}, \not{p}_-] \gamma_5}{2} + H_{1s}^\perp \frac{[\not{k}_T, \not{p}_-] \gamma_5}{2M_\Lambda} + i H_1^\perp \frac{[\not{k}_T, \not{p}_-]}{2M_\Lambda} \Big\} \\ & + \frac{M_\Lambda}{2P_\Lambda^-} \left\{ E - i E_s \gamma_5 + E_T^\perp \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T\sigma}}{M_\Lambda} \right. \\ & + D^\perp \frac{\not{k}_T}{M_\Lambda} + D_T' \epsilon_T^{\rho\sigma} \gamma_\rho S_{\Lambda T\sigma} + D_s^\perp \frac{\epsilon_T^{\rho\sigma} \gamma_\rho k_{T\sigma}}{M_\Lambda} \\ & + G_T' \gamma_5 \not{S}_{\Lambda T} + G_s^\perp \gamma_5 \frac{\not{k}_T}{M_\Lambda} + G^\perp \gamma_5 \frac{\epsilon_T^{\rho\sigma} \gamma_\rho k_{T\sigma}}{M_\Lambda} \\ & \left. + H_s \frac{[\not{p}_-, \not{p}_+] \gamma_5}{2} + H_T^\perp \frac{[\not{S}_{\Lambda T}, \not{k}_T] \gamma_5}{2M_\Lambda} + i H \frac{[\not{p}_-, \not{p}_+]}{2} \right\}, \end{aligned} \quad (21)$$

where the FFs on the r.h.s. depend on z and $k_T^2 \equiv -\mathbf{k}_T^2$. We use the shorthand notation [27] for the function G_{1s} :

$$G_{1s} = S_{\Lambda L} G_{1L} - \frac{k_T \cdot S_{\Lambda T}}{M_\Lambda} G_{1T} \quad (22)$$

and so forth for the other functions. It is easy to find that the decomposition of Δ is obtained from that of Φ by the replacements:

$$n_+ \leftrightarrow n_-, \quad \epsilon_T \rightarrow -\epsilon_T, \quad P^+ \rightarrow P_\Lambda^-, \quad M \rightarrow M_\Lambda, \quad x \rightarrow 1/z, \quad (23)$$

and the PDFs are replaced with the corresponding FFs (e.g. with f_1 replaced by D_1 and all other letters are capitalized).

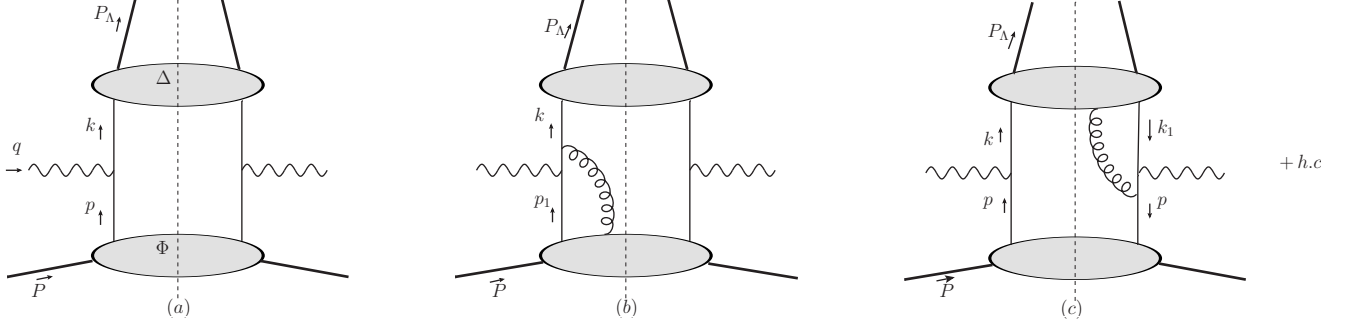


FIG. 2: Diagrams contributing to semi-inclusive DIS up to $\mathcal{O}(1/Q)$, the “h.c.” stands for the hermitian conjugation of (b) and (c) [41].

As shown in Eq. (21), there are three extra twist-3 T-odd FFs, denoted by E_T^\perp , D_T^\perp and G^\perp , which have not been presented in Refs. [27, 43]. These functions, analogous to the TMD PDFs e_T^\perp , f_T^\perp and g^\perp , arise due to the presence of the direction of the Wilson line n_+ in the unintegrated correlator (21). Among them, D_T^\perp is introduced to maintain the symmetry with other functions and simplify the expression of the results [26, 41]. As we have few experimental information on T-odd fragmentation functions, model calculation is an important way to acquire knowledge of these quantities, such as the spectator models [54–56]. In SIDIS, the contributions of the functions g^\perp and G^\perp have to be taken into account, and they could provide useful explanation for the difference of the A_{LU} and A_{UL} asymmetries [25]. Finally, E_T^\perp , D_T^\perp are polarized fragmentation functions appearing in polarized Λ hyperon production in SIDIS.

At last, we examine the quark-gluon-quark correlator $\tilde{\Phi}_A$ and $\tilde{\Delta}_A$ appearing in the last line of Eq. (11)

$$\tilde{\Phi}_A^\alpha(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} \int e^{ip \cdot \xi} \langle P | \bar{\psi}(0) \int_{\pm\infty}^{\xi^-} d\eta^- \mathcal{U}_{(0,+\infty)}^{n-} g F_\perp^{+\alpha}(\eta) \mathcal{U}_{(\eta,\xi)}^{n-} \mathcal{U}_{(+\infty,\eta)}^{n-} \psi(\xi) | P \rangle \Big|_{\substack{\eta^+ = \xi^+ = 0 \\ \eta_T = \xi_T}}, \quad (24)$$

$$\begin{aligned} \tilde{\Delta}_A^\alpha(z, k_T) = & \frac{1}{2z} \sum_X \int \frac{d\xi^+ d^2\xi_T}{(2\pi)^3} \int e^{ik \cdot \xi} \langle 0 | \int_{\pm\infty}^{\xi^+} d\eta^+ \mathcal{U}_{(+\infty,\eta)}^{n+} \\ & \times g F_\perp^{-\alpha}(\eta) \mathcal{U}_{(\eta,\xi)}^{n+} \psi(\xi) | P_\Lambda, S_\Lambda; X \rangle \langle P_\Lambda, S_\Lambda; X | \bar{\psi}(0) \mathcal{U}_{(0,+\infty)}^{n+} | 0 \rangle \Big|_{\substack{\eta^- = \xi^- = 0 \\ \eta_T = \xi_T}}. \end{aligned} \quad (25)$$

Compared to Φ and Δ , $\tilde{\Phi}_A^\alpha$ and $\tilde{\Delta}_A^\alpha$ contain an additional gluon leg [41–43, 57]. $\tilde{\Phi}_A^\alpha$ can be decomposed as

$$\tilde{\Phi}_A^\alpha(x, p_T) = \frac{xM}{2} \left\{ [(\tilde{f}^\perp - i\tilde{g}^\perp) \frac{p_{T\rho}}{M}] (g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho} \gamma_5) + (\tilde{h} + i\tilde{e}) i\gamma_T^\alpha + \cdots (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho} \gamma_5) \right\} \frac{\not{p}_+}{2}, \quad (26)$$

where we consider the target nucleon to be unpolarized. A complete parameterization of $\tilde{\Phi}_A^\alpha$ has given in Eq (3.45) of Ref. [41].

After performing the replacements in Eq. (23), we can also decompose the quark-gluon-quark correlator $\tilde{\Delta}_A^\alpha$ as follows

$$\begin{aligned} \tilde{\Delta}_A^\alpha(z, k_T) = & \frac{M_\Lambda}{2z} \left\{ [(\tilde{D}^\perp - i\tilde{G}^\perp) \frac{k_{T\rho}}{M_\Lambda} + (\tilde{D}'_T + i\tilde{G}'_T) \epsilon_{T\rho\sigma} S_{\Lambda T}^\sigma + (\tilde{D}_s^\perp + i\tilde{G}_s^\perp) \frac{\epsilon_{T\rho\sigma} k_T^\sigma}{M_\Lambda}] (g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho} \gamma_5) \right. \\ & \left. - (\tilde{H}_s + i\tilde{E}_s) \gamma_T^\alpha \gamma_5 + [(\tilde{H} + i\tilde{E}) - (\tilde{H}_T^\perp - i\tilde{E}_T^\perp) \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T}^\sigma}{M_\Lambda}] i\gamma_T^\alpha + \cdots \right\} \frac{\not{p}_-}{2}. \end{aligned} \quad (27)$$

It is convenient to parameterize the following combinations as

$$\frac{z}{2M_\Lambda} \text{Tr}[\tilde{\Delta}_{A\alpha} \sigma^{\alpha-}] = \tilde{H} + i\tilde{E} - \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{\Lambda T}^\sigma}{M_\Lambda} (\tilde{H}_T^\perp - i\tilde{E}_T^\perp), \quad (28)$$

$$\frac{z}{2M_\Lambda} \text{Tr}[\tilde{\Delta}_{A\alpha} i\sigma^{\alpha-} \gamma_5] = \tilde{H}_s + i\tilde{E}_s, \quad (29)$$

$$\begin{aligned} \frac{z}{2M_\Lambda} \text{Tr}[\tilde{\Delta}_{A\rho}(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5)\gamma^-] &= \frac{k_T^\alpha}{M_\Lambda}(\tilde{D}^\perp - i\tilde{G}^\perp) + \epsilon_T^{\alpha\rho}S_{\Lambda T\rho}(\tilde{D}_T + i\tilde{G}_T) \\ &+ \frac{\epsilon_T^{\alpha\rho}k_{T\rho}}{M_\Lambda}(\tilde{D}_s^\perp + i\tilde{G}_s^\perp), \end{aligned} \quad (30)$$

where the indices α, ρ and σ are restricted to be transverse, and we have used the combinations

$$\tilde{D}_T(z, k_T^2) = \tilde{D}'_T(z, k_T^2) - \frac{k_T^2}{2M_\Lambda^2}\tilde{D}_T^\perp(z, k_T^2), \quad (31)$$

$$\tilde{G}_T(z, k_T^2) = \tilde{G}'_T(z, k_T^2) - \frac{k_T^2}{2M_\Lambda^2}\tilde{G}_T^\perp(z, k_T^2), \quad (32)$$

Using the parameterizations of correlators in Eq. (11) and the above identities, we can calculate the hadronic tensor in the process in which a lepton scatters off an unpolarized target producing a polarized Λ hyperon. The complete results for the symmetric and antisymmetric part of the hadronic tensor are

$$\begin{aligned} 2MW_S^{\mu\nu} = & 2z \int d^2p_T d^2k_T \delta^2(\mathbf{p}_T - \mathbf{k}_T + \mathbf{q}_T) \\ & \times \left\{ -g_\perp^{\mu\nu}[f_1 D_1 + \frac{\epsilon_\perp^{\rho\sigma}k_{\perp\rho}S_{\Lambda\perp\sigma}}{M_\Lambda}f_1 D_{1T}^\perp] - \frac{(p_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{\Lambda\perp\rho} + S_{\Lambda\perp}^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho})}{2M}h_1^\perp H_{1T} \right. \\ & - \frac{(p_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho} + k_\perp^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho})}{2MM_\Lambda}h_1^\perp H_{1s}^\perp - (-g_\perp^{\mu\nu}(k_\perp \cdot p_\perp) + k_\perp^{\{\mu}p_\perp^{\nu\}})\frac{1}{MM_\Lambda}h_1^\perp H_1^\perp \\ & + \frac{t^{\{\mu}k_\perp^{\nu\}}}{Q}[\frac{2f_1\tilde{D}^\perp}{z} + \frac{2xM}{M_\Lambda}hH_1^\perp] + \frac{t^{\{\mu}p_\perp^{\nu\}}}{Q}[\frac{M_\Lambda}{M}\frac{2\tilde{H}}{z}h_1^\perp + 2xf^\perp D_1] \\ & + \frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{\Lambda\perp\rho}}{Q}[-\frac{p_\perp \cdot k_\perp}{M}2h_1^\perp \frac{\tilde{H}_T^\perp}{z} + \frac{p_\perp \cdot k_\perp}{M_\Lambda}2xf^\perp D_{1T}^\perp + M_\Lambda f_1 \frac{2\tilde{D}'_T}{z} - 2xMhH_{1T}] \\ & + \frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho}}{Q}[S_{\Lambda L}2f_1 \frac{\tilde{D}_L^\perp}{z} - \frac{2(S_{\Lambda\perp} \cdot k_\perp)}{M_\Lambda}f_1 \frac{\tilde{D}_T^\perp}{z} - S_{\Lambda L} \frac{M}{M_\Lambda}2xhH_{1L}^\perp + \frac{(S_{\Lambda\perp} \cdot k_\perp)M}{M_\Lambda^2}2xhH_{1T}^\perp \\ & + \frac{(S_{\Lambda\perp} \cdot p_\perp)}{M}2h_1^\perp \frac{\tilde{H}_T^\perp}{z} - \frac{(S_{\Lambda\perp} \cdot p_\perp)}{M_\Lambda}2xD_{1T}^\perp f^\perp] \\ & \left. + \frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho}}{Q}[\frac{(S_{\Lambda\perp} \cdot k_\perp)}{M}2h_1^\perp \frac{\tilde{H}_T}{z} - 2S_{\Lambda L} \frac{M_\Lambda}{M}h_1^\perp \frac{\tilde{H}_L}{z} + \frac{(S_{\Lambda\perp} \cdot k_\perp)}{M_\Lambda}2xg^\perp G_{1T} - 2S_{\Lambda L}xg^\perp G_{1L}]\right\}, \quad (33) \end{aligned}$$

and

$$\begin{aligned} 2MW_A^{\mu\nu} = & 2z \int d^2p_T d^2k_T \delta^2(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T) \\ & \times \left\{ i\epsilon_\perp^{\mu\nu}f_1 G_{1s} \right. \\ & + i\frac{t^{\{\mu}k_\perp^{\nu\}}}{Q}[-2f_1 \frac{\tilde{G}^\perp}{z} - \frac{xM}{M_\Lambda}2eH_1^\perp] + i\frac{t^{\{\mu}p_\perp^{\nu\}}}{Q}[2\frac{M_\Lambda}{zM}\tilde{E}h_1^\perp + 2xg^\perp D_1] \\ & + i\frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}S_{\Lambda\perp\rho}}{Q}[\frac{p_\perp \cdot k_\perp}{M_\Lambda}2xg^\perp D_{1T}^\perp + \frac{p_\perp \cdot k_\perp}{M}2h_1^\perp \frac{\tilde{E}_T^\perp}{z} + 2MxeH_{1T} + M_\Lambda 2f_1 \frac{\tilde{G}'_T}{z}] \\ & + i\frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}k_{\perp\rho}}{Q}[2S_{\Lambda L}f_1 \frac{\tilde{G}_L^\perp}{z} - \frac{(S_{\Lambda\perp} \cdot k_\perp)}{M_\Lambda}2f_1 \frac{\tilde{G}_T^\perp}{z} + 2S_{\Lambda L} \frac{M}{M_\Lambda}xeH_{1L}^\perp - \frac{(S_{\Lambda\perp} \cdot k_\perp)M}{M_\Lambda^2}2xeH_{1T}^\perp \\ & - \frac{p_\perp \cdot S_{\Lambda\perp}}{M_\Lambda}2xg^\perp D_{1T}^\perp - \frac{(S_{\Lambda\perp} \cdot p_\perp)}{M}2h_1^\perp \frac{\tilde{E}_T^\perp}{z}] \\ & \left. + i\frac{t^{\{\mu}\epsilon_\perp^{\nu\}\rho}p_{\perp\rho}}{Q}[-2\frac{S_{\Lambda L}M_\Lambda}{M}h_1^\perp \frac{\tilde{E}_L}{z} + \frac{(S_{\Lambda\perp} \cdot k_\perp)}{M}2h_1^\perp \frac{\tilde{E}_T}{z} + 2S_{\Lambda L}xf^\perp G_{1L} - \frac{k_\perp \cdot S_{\Lambda\perp}}{M_\Lambda}2xf^\perp G_{1T}]\right\}. \quad (34) \end{aligned}$$

III. THE RESULT OF STRUCTURE FUNCTIONS

By assuming single photon exchange, the cross section of Λ hyperon production can be written in terms of the following structure functions:

$$\begin{aligned} \frac{d\sigma}{dx dy dz d\phi d\psi dP_{h\perp}^2} = & \frac{\alpha^2}{xyQ^2} \left\{ (1-y + \frac{1}{2}y^2)F_{UUU} + (2-y)\sqrt{1-y}\cos\phi_\Lambda F_{UUU}^{\cos\phi_\Lambda} \right. \\ & + (1-y)\cos(2\phi_\Lambda)F_{UUU}^{\cos 2\phi_\Lambda} + \lambda y\sqrt{(1-y)}\sin\phi_\Lambda F_{LUU}^{\sin\phi_\Lambda} \\ & + S_{\Lambda||}[(2-y)\sqrt{1-y}\sin\phi_\Lambda F_{UUL}^{\sin\phi_\Lambda} + (1-y)\sin 2\phi_\Lambda F_{UUL}^{\sin 2\phi_\Lambda}] \\ & + S_{\Lambda||}\lambda[y(1-\frac{1}{2}y)F_{LUL} + y\sqrt{1-y}\cos\phi_\Lambda F_{LUL}^{\cos\phi_\Lambda}] \\ & + |S_{\Lambda\perp}|[(1-y + \frac{1}{2}y^2)\sin(\phi_\Lambda - \phi_{S_\Lambda})F_{UUT}^{\sin(\phi_\Lambda - \phi_{S_\Lambda})} + (1-y)\sin(\phi_\Lambda + \phi_{S_\Lambda})F_{UUT}^{\sin\phi_\Lambda + \phi_{S_\Lambda}} \\ & + (1-y)\sin(3\phi_\Lambda - \phi_{S_\Lambda})F_{UUT}^{\sin(3\phi_\Lambda - \phi_{S_\Lambda})} + (2-y)\sqrt{1-y}\sin(\phi_{S_\Lambda})F_{UUT}^{\sin\phi_{S_\Lambda}} \\ & + (2-y)\sqrt{1-y}\sin(2\phi_\Lambda - \phi_{S_\Lambda})F_{UUT}^{\sin(2\phi_\Lambda - \phi_{S_\Lambda})}] \\ & + |S_{\Lambda\perp}|\lambda[y(1-\frac{1}{2}y)\cos(\phi_\Lambda - \phi_{S_\Lambda})F_{LUT}^{\cos(\phi_\Lambda - \phi_{S_\Lambda})} + y\sqrt{(1-y)}\cos(\phi_{S_\Lambda})F_{LUT}^{\cos\phi_{S_\Lambda}} \\ & \left. + y\sqrt{(1-y)}\cos(2\phi_\Lambda - \phi_{S_\Lambda})F_{LUT}^{\cos(2\phi_\Lambda - \phi_{S_\Lambda})} + \dots \right\}, \end{aligned} \quad (35)$$

where $F_{ABC} = F_{ABC}(x, z, P_{h\perp}^2)$, and the ellipsis denotes the terms that are not considered in this study. The subscripts A, B and C indicate the polarizations of the incoming lepton, the target nucleon and the produced Λ hyperon, respectively. Our definitions are consistent with those in Ref. [28]. We use U, L and T to denote unpolarized, longitudinally and transversely polarized particles, where “L” (longitudinal) or “T” (transverse) means that the spin vector parallel to or is perpendicular to the virtual photon axis [27, 58]. The depolarization factors in front of the structure functions come from the contraction of the leptonic tensor $L^{\mu\nu}$ with the tensor structures appearing in the hadronic tensor.

Substituting Eqs. (19, 21, 26, 27) into Eq. (11) and applying Eqs. (8, 35), We obtain the expressions of the structure functions as follows

$$F_{UUT}^{(\phi_\Lambda - \phi_{S_\Lambda})} = \mathcal{I}\left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_\Lambda} f_1 D_{1T}^\perp\right] \quad (36)$$

$$F_{UUL}^{\sin 2\phi_\Lambda} = \mathcal{I}\left[-\frac{2(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\hat{\mathbf{h}} \cdot \mathbf{k}_T) - \mathbf{p}_T \cdot \mathbf{k}_T}{MM_\Lambda} h_1^\perp H_{1L}^\perp\right] \quad (37)$$

$$F_{UUT}^{\sin(\phi_\Lambda + \phi_{S_\Lambda})} = \mathcal{I}\left[-\frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} h_1^\perp H_1\right] \quad (38)$$

$$F_{UUT}^{\sin(3\phi_\Lambda - \phi_{S_\Lambda})} = \mathcal{I}\left[-\frac{4(\hat{\mathbf{h}} \cdot \mathbf{p}_T)(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - \mathbf{k}_T^2(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - 2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\mathbf{k}_T \cdot \mathbf{p}_T)}{2MM_\Lambda^2} h_1^\perp H_{1T}^\perp\right] \quad (39)$$

$$F_{LUL} = \mathcal{I}[f_1 G_{1L}] \quad (40)$$

$$F_{LUT}^{\cos(\phi_\Lambda - \phi_{S_\Lambda})} = \mathcal{I}\left[\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_\Lambda} f_1 G_{1T}\right] \quad (41)$$

$$F_{UUL}^{\sin\phi_\Lambda} = \frac{2M}{Q}\mathcal{I}\left\{\frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_\Lambda}\left(\frac{M_\Lambda}{M}f_1\frac{\tilde{D}_L^\perp}{z} - xhH_{1L}^\perp\right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M}\left(\frac{M_\Lambda}{M}h_1^\perp\tilde{H}_L + xg^\perp G_{1L}\right)\right\} \quad (42)$$

$$\begin{aligned} F_{UUT}^{\sin(2\phi_\Lambda - \phi_{S_\Lambda})} = & \frac{2M}{Q}\mathcal{I}\left\{\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - \mathbf{k}_T^2}{2M_\Lambda^2}\left(\frac{M_\Lambda}{M}f^\perp\frac{\tilde{D}_T^\perp}{z} - xhH_{1T}^\perp\right) \right. \\ & \left. + \frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2MM_\Lambda}[(xf^\perp D_{1T}^\perp - \frac{M_\Lambda}{M}h_1^\perp\frac{\tilde{H}_T^\perp}{z}) - (\frac{M_\Lambda}{M}h_1^\perp\frac{\tilde{H}_T}{z} + xg^\perp G_{1T})]\right\} \end{aligned} \quad (43)$$

$$F_{UUT}^{\sin\phi_{S_\Lambda}} = \frac{2M}{Q}\mathcal{I}\left\{\left(\frac{M_\Lambda}{M}f_1\frac{\tilde{D}_T}{z} - xhH_1\right) + \frac{\mathbf{k}_T \cdot \mathbf{p}_T}{2MM_\Lambda}\left[\left(\frac{M_\Lambda}{M}h_1^\perp\tilde{H}_T^\perp - xf_1 D_{1T}^\perp\right) - \left(\frac{M_\Lambda}{M}h_1^\perp\frac{\tilde{H}_T}{z} - xg^\perp G_{1T}\right)\right]\right\} \quad (44)$$

$$F_{LUT}^{\cos \phi_{S\Lambda}} = \frac{2M}{Q} \mathcal{I} \left\{ - \left(\frac{M_\Lambda}{M} f_1 \frac{\tilde{G}_T}{z} + x e H_1 \right) + \frac{\mathbf{p}_T \cdot \mathbf{k}_T}{2M M_\Lambda} \left[\left(\frac{M_\Lambda}{M} h_1^\perp \frac{\tilde{E}_T^\perp}{z} + x g^\perp D_{1T}^\perp \right) + \left(\frac{M_\Lambda}{M} h_1^\perp \frac{\tilde{E}_T}{z} - x f^\perp G_{1T} \right) \right] \right\} \quad (45)$$

$$F_{LUT}^{\cos(2\phi_\Lambda - \phi_{S\Lambda})} = \frac{2M}{Q} \mathcal{I} \left\{ - \frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)^2 - \mathbf{k}_T^2}{2M_\Lambda^2} \left(\frac{M_\Lambda}{M} f_1 \frac{\tilde{G}_T^\perp}{z} + x e H_{1T}^\perp \right) - \frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_T)(\hat{\mathbf{h}} \cdot \mathbf{p}_T) - \mathbf{k}_T \cdot \mathbf{p}_T}{2M M_\Lambda} \left[\left(x g^\perp D_{1T}^\perp + \frac{M_\Lambda}{M} h_1^\perp \frac{\tilde{E}_T^\perp}{z} \right) - \left(\frac{M_\Lambda}{M} h_1^\perp \frac{\tilde{E}_T}{z} - x f^\perp G_{1T} \right) \right] \right\} \quad (46)$$

$$F_{LUL}^{\cos \phi_\Lambda} = \frac{2M}{Q} \mathcal{I} \left\{ \frac{\hat{\mathbf{h}} \cdot \mathbf{p}_T}{M} \left(\frac{M_\Lambda}{M} h_1^\perp \frac{\tilde{E}_L}{z} - x f^\perp G_{1L} \right) - \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_\Lambda} \left(\frac{M_\Lambda}{M} f_1 \frac{\tilde{G}_L^\perp}{z} + x e H_{1L}^\perp \right) \right\} \quad (47)$$

where the structure functions depend on the polarization of the lepton (U and L) and produced Λ hyperon (L and T). In the expressions of the structure functions we introduce the normalized vector $\hat{\mathbf{h}} = \mathbf{P}_{h\perp}/|\mathbf{P}_{h\perp}|$ and the convolution integral

$$\mathcal{I}[\omega f D] = x \sum_a e_a^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{\Lambda\perp}/z) \omega(\mathbf{p}_T, \mathbf{k}_T) f^a(x, p_T^2) D^a(z, k_T^2), \quad (48)$$

Here, $\omega(\mathbf{p}_T, \mathbf{k}_T)$ is an arbitrary function and the summation runs over quarks and anti-quarks. We have not included the unpolarized Λ terms since those have been presented in Ref. [41]. In above results, we have applied the equation of motion relations [27] among the twist-2 and twist-3 TMD PDFs and FFs. Thus we can find a feature that PDFs and FFs do not appear in a symmetric fashion: there are only twist-3 PDFs without tilde and twist-3 FFs with a tilde. This asymmetry is because in Eq. (35) the structure functions are introduced by using an asymmetric way in the $\gamma^* N$ collinear frames rather than the ΛN collinear frames.

In the following, we will briefly discuss our results.

1. The contributions of the new polarized FFs \tilde{D}_T^\perp , \tilde{E}_T^\perp have to be taken into account in the calculation of the structure functions. Our calculation shows that \tilde{D}_T^\perp appear in the $\phi_{S\Lambda}$ -dependent structure functions $F_{UUT}^{(2\phi_\Lambda - \phi_{S\Lambda})}$, while \tilde{E}_T^\perp appears in $F_{LUT}^{\cos \phi_{S\Lambda}}$ and $F_{LUT}^{(2\cos\Lambda - \cos\phi_{S\Lambda})}$. The twist-3 TMDs g^\perp gives contributions to all the twist-3 structure functions except $F_{LUL}^{\cos \phi_{S\Lambda}}$.

2. The leading twist structure functions in our results are corresponding to Table. III of Ref. [28] and Eq.(38) in Ref. [30]. The six twist-3 structure functions in our result show some similarity compared with the expressions in Ref. [41], while the role of PDFs and FFs have being reversed in the former.

3. For the production of unpolarized Λ hyperon, the structure functions were discussed in Ref. [41]. On the other hand, our results show that the polarized FFs play important roles in the quark fragmenting to Λ hyperon, i.e., the transversely polarized structure functions $F_{UUT}^{\sin(2\phi_\Lambda - \phi_{S\Lambda})}$, $F_{UUT}^{\sin \phi_\Lambda}$, $F_{UUT}^{\sin \phi_{S\Lambda}}$ and $F_{LUT}^{\cos \phi_{S\Lambda}}$ all contain the spin-dependent T-odd functions which have been studied in Refs. [59, 60]. Because the TMD FFs are found to be universal in different different process [32, 61], the FFs can be also applied to study the Λ production in $e^+ e^-$ annihilation process.

At last, we perform the integration over the transverse momentum $P_{\Lambda\perp}$ for the structure functions given above, the non-vanished integrated structure functions are as follows [28]

$$F_{LUL}(x, z) = x \sum_a e_a^2 f_1(x) G_1(z), \quad (49)$$

$$F_{UUT}^{\sin \phi_{S\Lambda}}(x, z) = x \sum_a e_a^2 \frac{2M_\Lambda}{Q} f_1(x) \frac{\tilde{D}_T(z)}{z}, \quad (50)$$

$$F_{LUT}^{\cos \phi_{S\Lambda}}(x, z) = -x \sum_a e_a^2 \frac{2M}{Q} \left(\frac{M_\Lambda}{M} f_1(x) \frac{\tilde{G}_T(z)}{z} + x e(x) H_1(z) \right), \quad (51)$$

where the functions on r.h.s are given by

$$f_1(x) = \int d^2 \mathbf{p}_T f_1(x, p_T^2), \quad D_1(z) = z^2 \int d^2 \mathbf{k}_T D_1(z, k_T^2), \quad (52)$$

and so forth for the other functions. In Eq. (50), there is no contribution from the distribution h , since T-odd PDFs vanish under time reversal.

$$\int d^2 \mathbf{p}_T h(x, p_T^2) = 0. \quad (53)$$

The contribution from $\tilde{D}_T(z)$ still remains due to the final-state interaction effects [41, 58] during fragmentation. Thus, measurement of the structure function $F_{UUT}^{\sin\phi_{S\Lambda}}(x, z)$ provides a unique opportunity to explore the T-odd FFs $\tilde{D}_T(z)$. Finally, the structure function F_{LUL} is associated with the longitudinal spin transfer from longitudinally polarized lepton beam to the Λ hyperon through the convolution of $f_1(x)$ and $G_1(z)$ [3, 4, 27].

IV. CONCLUSION

SIDIS has been recognized as an very useful tool to study the spin structure of Λ hyperon. In this work, we have studied the production of polarized Λ hyperon by unpolarized or longitudinally lepton beam scattered off an unpolarized nucleon target. Using the complete decomposition of the parton correlation functions for fragmentation up to twist-three, we have presented the tree-level result of the cross-section for the process $\ell + N \rightarrow \ell' + \Lambda + X$ at order $1/Q$, based on the TMD framework. We find that, among a total of twelve polarized $P_{\Lambda\perp}$ -dependent structure functions which depends, seven of them are at twist-three level and can be expressed as convolutions of twist-two and twist-three TMD PDFs and FFs. We give the complete expressions for these structure functions, for each of which there are several twist-three functions that contribute. In our analysis we also include the T-odd TMD PDFs g^\perp and FFs \tilde{E}_T^\perp and \tilde{D}_T^\perp , which were not taken into account in previous studies and may contribute in spin asymmetries in polarized Λ production. The measurements of $\ell + N \rightarrow \ell' + \Lambda + X$ thus can provide useful observables to understand the fragmentation mechanism and polarization phenomena of the Λ hyperon.

Acknowledgements

This work is partially supported by the National Natural Science Foundation of China (Grants No. 11575043 and No. 11120101004), and by the Qing Lan Project

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